Path-averaged optical soliton in double-periodic dispersion-managed systems

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A path-averaged Gabitov-Turitsyn model governing optical signal propagation down the dispersion-managed (DM) transmission line is studied numerically. A different numerical algorithm to find a soliton solution for an arbitrary periodic DM system is proposed. Applying developed technique we analyze soliton solutions for few important practical systems.

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I. INTRODUCTION

Dispersion management is a key technique in high-bit-rate optical data transmission. The dispersion-managed (DM) transmission systems use a periodic alternation of positive and negative dispersion fiber pieces. The aim of dispersion management is to minimize path-averaged dispersion of line while keeping high enough local dispersion. In the linear regime, total compensation of the path-averaged dispersion would lead to the total recovery of the signal. However, in nonlinear propagation regimes an arbitrary pulse cannot be completely recovered at the ends of periodic sections. This is possible only for pulses of the special form—DM solitons. In this paper, we construct such special pulses for a range of double-periodic DM systems within the path-averaged model [1].

II. PATH-AVERAGED MODEL IN SPECTRAL DOMAIN

The propagation of an optical pulse in a DM fiber line is described by a normalized nonlinear Schrödinger equation (NLSE) with periodically varying coefficients,

$$iA_z + d(z)A_{tt} + c(z)|A|^2 A = 0, \tag{1}$$

where A is a scaled envelope, periodic function $d(z)$ with the period T_1 is a dispersion of the line (a dispersion map) and T_2-periodic function $c(z)$ describes power oscillations due to loss or gain variations in the line. Discussion of the physical effects leading to Eq. (1) and typical parameters can be found in Ref. [2]. We follow here notations and normalizations introduced in Ref. [2]: z is normalized to a length Z_0 (in km) defined below; time t is measured in some time constant t_0 (in ps) that can be specified for each specific problem; an envelope of the electric field E is normalized to the power parameter P_0 (in W), $|E|^2 = P_0|A|^2\exp[-2\gamma(z-z_k)]$ for $z_k \leqslant z < z_{k+1} = z_k + Z_a/Z_0$, where γ is a coeffecient of a fiber loss. Function $d(z) = \tilde{d}(z) + \langle d\rangle = -\beta_2(z)Z_0/(2t_0^2)$ describes periodic compensation of dispersion (with the period L in physical units), where β_2 is the first-order group velocity dispersion. Periodic (with the period Z_a in real world units) function $c(z) = \tilde{c}(z) + \langle c\rangle = 2\pi n_2 P_0 Z_0 \exp[-2\gamma(z-z_k)]/(\lambda_0 A_{eff})$ for $z_k \leqslant z < z_{k+1} = z_k + Z_a/Z_0$ describes the power variation due to fibre loss and amplifier gain that is accounted through transformation of the pulse power at junctions corresponding to the locations z_k of the optical amplifiers, where n_2 is the nonlinear refraction index, $\lambda_0 = 1.55\ \mu$m is the carrier wavelength, A_{eff} is the effective fiber area. The amplification distance Z_a in general can be different from the compensation period L. We consider a general case when L and Z_a are rational commensurable, namely, $nZ_a = mL = Z_0$ with integers n and m. This includes as particular limits all known and studied cases and allows us to describe a regime with short-scale ($L \ll Z_a$) management. The distance $z = Z/Z_0$ is normalized in Eq. (1) by a minimal common period Z_0 of the functions d and c and the averaging throughout the paper is over this period. In the normalized units periodic d and c have basic periods $T_1 = 1/m$ and $T_2 = 1/n$, respectively.

If a characteristic nonlinear length of the pulse is larger than the period of the dispersion variations then one can apply an averaging approach to simplify the basic equation. The resulting path-averaged Gabitov-Turitsyn equation [1] presented in the spectral domain takes the following form:

$$i\Psi_z - \langle d\rangle \omega^2 \Psi + G(\Psi,\omega) = 0, \tag{2}$$

where $\Psi(\omega_k)$ is a Fourier transformation of an averaged variable, $\langle d\rangle = \int_0^1 d(z)dz$ is an average dispersion, and $G(\Psi,\omega)$ is a nonlinear integral operator,

$$G(\Psi,\omega) = \int T_{\omega 123}\Psi^*(\omega_1)\Psi(\omega_2)\Psi(\omega_3)\delta \times(\omega+\omega_1-\omega_2-\omega_3)d\omega_1 d\omega_2 d\omega_3, \tag{3}$$

with a matrix element $T_{\omega 123}$ that is a complex function of a specific combination $\Delta\Omega=\omega^2+\omega_1^2-\omega_2^2-\omega_3^2$,

$$T_{\omega 123}=T(\Delta\Omega)=\int_0^1 c(z)e^{iR(z)\Delta\Omega}dz. \tag{4}$$

A function $R(z)$ is defined from equation $R_z=d(z)-\langle d\rangle$. Equation (2) has a standard form typical for models describing a four-wave interaction with quadratic dispersion law. Specific properties of the model are then given by a dependence of the matrix element T on $\Delta\Omega$. In the case of the so-called weak dispersion management [''small'' overall effect of a variation of $R(z)$] Eq. (2) can be reduced to NLSE with constant coefficients [3]. In this paper, we construct numerically exact periodic solutions (DM solitons) of the path-averaged model (2) for a range of practical DM lines.

A. Soliton solution

Similar to the well-studied NLSE ($T=1$), we seek a soliton solution of Eq. (2) in the form $\Psi(\omega,z)=\psi(\omega)\exp(i\lambda^2 z)$. An equation for a DM soliton shape $\psi(\omega)$ takes a form

$$(\lambda^2+\langle d\rangle\omega^2)\psi=G(\psi,\omega). \tag{5}$$

Note that for a real matrix element T one can find solution of this equation with a real function $\psi(\omega)$. In general case the matrix element T is a real function only for partial subclasses of transmission systems and corresponding coefficients $d(z)$ and $c(z)$. The well-known and studied case of the real matrix element is a lossless two-step DM system with matrix element $T=c_0\sin(s\Delta\Omega)/(s\Delta\Omega)$, where s is a map strength and $c(z)=c_0$ [4]. Another important example of the real matrix element is a long-scale dispersion-managed line [5]. Note, however, that for a short-scale dispersion management the corresponding matrix element is complex. Here we say the long or short scale if the length of the dispersion management is greater or smaller than the distance between amplifiers.

An effective numerical method to find a soliton solution for Eq. (5) was proposed by Petviashvili [6,7] and was applied to DM soliton problems in Refs. [8–10]. A stabilizing factor of this iteration method can be found easily because the right-hand size of the Eq. (5) is a homogeneous function with degree 3. See for details Ref. [9]. Iterations used in the Petviashvili's method request a computation of the integral operator $G(\psi,\omega)$. After single integration using δ function, the operator $G(\psi,\omega)$ includes a double integration, therefore, in general a computation of $G(\psi,\omega)$ requests N^3 operations, where N is a number of points. Some advanced approaches have been developed in Ref. [10,21]. Here we apply an effective numerical algorithm to solve this problem. The idea of our method is based on an approximation of the matrix element $T(\Delta\Omega)$ by an appropriate set of functions. This approximation allows us to apply a fast computation of convolutions and to reduce a number of operations to $MN\log_2(N)$, where M depends on the approximation of $T(\Delta\Omega)$. The main goal of this paper is to study properties of the path-averaged DM solitons in few practical systems using proposed method.

B. Approximations of the matrix element

If the matrix element $T(\Delta\Omega)$ is a polynomial or might be approximated by polynomial then a computation of the integral operator G can be reduced to computation of a sequence of the correlations and the number of operations is equal to $(S^2/2)N\log_2(N)$, where S is a power of the polynomial. Consider a partial form of the matrix element $T(\Delta\Omega)=(\Delta\Omega)^p$. Integrating by ω_3 we eliminate δ function,

$$\int [\omega^2+\omega_1^2-\omega_2^2-(\omega+\omega_1-\omega_2)^2]^p\psi^*(\omega_1)\psi(\omega_2) \times\psi(\omega+\omega_1-\omega_2)d\omega_1 d\omega_2. \tag{6}$$

Using the identity $\omega^2+\omega_1^2-\omega_2^2-(\omega+\omega_1-\omega_2)^2=2(\omega_1-\omega_2)(\omega_1-\omega)$ and introducing a new variable $x=\omega_1-\omega_2$ we get

$$2^p\int x^p(\omega_2+x-\omega)^p\psi^*(\omega_2+x)\psi(\omega_2)\psi(\omega+x)d\omega_2 dx. \tag{7}$$

Using the Newton formula we obtain

$$2^p\int x^p\sum_{k=0}^{p}C_p^k(-\omega)^{p-k}(\omega_2+x)^k\psi^*(\omega_2+x)\psi(\omega_2) \times\psi(\omega+x)d\omega_2 dx. \tag{8}$$

Denoting $f_k(x)=\int(\omega_2+x)^k\psi^*(\omega_2+x)\psi(\omega_2)d\omega_2$ we rewrite

$$2^p\int x^p\sum_{k=0}^{p}C_p^k(-\omega)^{p-k}f_k(x)\psi(\omega+x)dx \tag{9}$$

$$=2^p\sum_{k=0}^{p}C_p^k(-\omega)^{p-k}g_{pk}(\omega), \tag{10}$$

where $g_{pk}=\int x^p f_k(x)\psi(\omega+x)dx$.

It is seen that one has to compute $p+1$ integrals f_k and $p+1$ integrals g_{pk}. Thus, we can estimate a number of operation as $(S^2/2)N\log_2(N)$ for an arbitrary polynomial of a power S.

This computation method can be used for some approximations of the initial Eq. (2) [11,12]. In this model the matrix element $T(\Delta\Omega)$ is approximated by the quadratic polynomial $T(\Delta\Omega)=T(0)+T''(0)[(\Delta\Omega)^2/2]$, that is, the two first terms of the Taylor series.

III. PATH-AVERAGED MODEL IN TIME DOMAIN

A. Averaged equation

Here we briefly recall previously obtained results on averaging of a double-periodic NLSE using slightly different approach [5]. Basic model reads

$$iA_z+d(z)A_{tt}+c(z)|A|^2A=0, \tag{11}$$

here the functions $d(z)$ and $c(z)$ are periodic $d(z)=d(z+T_1)$ and $c(z)=c(z+T_2)$.

The first transform is called Floquet-Lyapunov one and eliminates the periodicity of $d(z)$,

$$A(t,z)=e^{iR(z)D^2}B(t,z),\quad \frac{dR}{dz}=d(z)-\langle d\rangle,\quad D:=\frac{\partial}{\partial t},$$

$$\langle f\rangle:=\int_0^1 f(z)dz. \tag{12}$$

The equation takes the form

$$iB_z=-\langle d\rangle D^2B-c(z)e^{-iR(z)D^2}[|e^{iR(z)D^2}B|^2e^{iR(z)D^2}B]. \tag{13}$$

The right-hand side (rhs) is periodic and we are able to average this equation. To average Eq. (13) in a correct way the rhs should be small or Eq. (13) should have the so-called Bogolyubov standard form [13].

The operator $e^{iR(z)D^2}$ and its inverse one in the nonlinear term are bounded for any real function $R(z)$ because in Fourier space it is an exponential factor $e^{-iR(z)\omega^2}$. Therefore, we assume that $c(z)$ is small function. In the linear term we have a combination $\langle d\rangle D^2$ and therefore we can assume that this combination is a small operator. Using a small parameter ε we specify the parameter $\langle d\rangle$, the function $c(z)$, and the operator D^2 to have the following scaling:

$$\langle d\rangle D^2\sim\varepsilon,\quad c(z)\sim\varepsilon. \tag{14}$$

The first relation means that a soliton is not very narrow for $\langle d\rangle\neq 0$. The second condition means that the nonlinearity is weak. Further, we imply these scales for a notation $O(\varepsilon)$ and similar.

Next we apply the so-called Bogolyubov-Krylov transform that eliminates the periodic part of the nonlinear terms. The resulting averaged equation is

$$iC_z+\langle d\rangle D^2C+N(C)=O(\varepsilon^2), \tag{15}$$

where

$$N(C)=\int_0^1 c(s)e^{-iR(s)D^2}\times[|e^{iR(s)D^2}C(z,t)|^2e^{iR(s)D^2}C(z,t)]ds. \tag{16}$$

The corresponding transform has the following form:

$$B=C+i\int_0^z\{c(s)e^{-iR(s)D^2}[|e^{iR(s)D^2}C(s,t)|^2e^{iR(s)D^2}C(s,t)]-N(C)\}ds. \tag{17}$$

B. Averaged operator

In the case of two-step dispersion map built from a piece of a fiber with dispersion $d_1+\langle d\rangle$ and lenght l_1 followed by a piece of a fiber with dispersion $d_2+\langle d\rangle$ and length $l_2=1-l_1$, the operator N takes the form

$$N(C)=\frac{1}{s}\int_{-s/2}^{s/2}S(y)e^{-iyD^2}[|e^{iyD^2}C(z,t)|^2e^{iyD^2}C(z,t)]dy, \tag{18}$$

where $s=d_1l_1$ and

$$S(y)=l_1c\left[\left(\frac{y}{s}+\frac{1}{2}\right)l_1\right]+(1-l_1)c\left[1-\left(\frac{y}{s}+\frac{1}{2}\right)(1-l_1)\right]. \tag{19}$$

For the Gabitov-Turitsyn model with $c(z)=c_0$, we get $S(y)=c_0$.

Note that from Eqs. (18) and (19) it is seen that if $S(y)$ is an even function then the matrix element T is a real function. In particular, if $l_1=1/2$ then $S(y)$ is the even function for any function $c(z)$ and therefore the matrix element T is real. Or in other words, without loss of generality, we can choose a free constant in definition of $R(z)$ to kill the imaginary part of the matrix element for the long-scale dispersion management with the equal two-piece elements.

C. Integral approximation

For a computation of integral operator $N(C)$, we apply direct method. The first step is to compute the integral with respect to s using some quadrature relations with weight coefficients W_m at points s_m

$$N(C)\approx\sum_{m=0}^{M}W_mc(s_m)e^{-iR(s_m)D^2}\times[|e^{iR(s_m)D^2}C(z,t)|^2e^{iR(s_m)D^2}C(z,t)]. \tag{20}$$

To compute the term $e^{iR(s_m)D^2}C(z,t)$ we apply the Fourier transform, then multiply by factor $e^{-iR(s_m)\omega^2}$, and make the inverse Fourier transform again. And the last step is to apply the operator $W_mc(s_m)e^{-iR(s_m)D^2}$ using the direct and inverse Fourier transform. This procedure requests $3M+1$ the Fourier transforms with $n\log_2(n)$ operations and $4n$ multiplications.

Also we remark that the function $R(s)$ is periodic and it is reasonable to choose s_m for equal values of $R(s)$. Then we decrease a number of operations by factor of 2.

IV. EXAMPLES OF COMPUTATION

We apply now our algorithm to find DM soliton solution for a general case with different periods of power and dispersion oscillations. We consider different matrix elements corresponding to practical fiber optical lines.

The simplest and important example of matrix element is $T(\Delta\Omega)=c_0\sin(s\Delta\Omega/2)/(s\Delta\Omega/2)$, where s is the dispersion map strength. This matrix element arises for lossless equation with two-step dispersion map.

Next we consider the fiber lines with different (but rational commensurable) periods of power and dispersion oscillations. Namely, we analyze two opposite limits: the short-scale dispersion management and long-scale two-step dispersion map.

For considered cases, Eq. (5) is characterized by six dimensionless parameters: λ, the averaged dispersion $\langle d\rangle$, the variation of the dispersion d, the nonlinearity parameter c_0, the loss parameter $G=\exp(2\gamma Z_a)$, and the ratio for the length of the dispersion map and the distance between the amplifiers L/Z_a. For our numeric computation we put $\lambda=c_0=1$ and choose $\alpha=0.21$ dB/km, $\gamma=0.05\alpha\ln 10$, $Z_a=40$ km for an evalutaion of the loss parameter G.

A. Long-scale management

First, we consider the case of a long-scale two-step dispersion management with $L\geqslant Z_a$. Let the distance between optical amplifiers be Z_a (km) and $L=2KZ_a$ (km), where $K=1,2,\ldots$. Dispersion $d(z)=d+\langle d\rangle$ if $0<z<K$ and $d(z)=-d+\langle d\rangle$ if $K<z<2K$. Mean-free function R defined above can be found as $R(z)=d(z-K/2)/2$ if $0<z<K$ and $R(z)=-d(z-3K/2)/2$ if $K<z<2K$. A function $c(z)$ is $c(z)=c_0\exp(-2\gamma z)$ if $0<z<1$. The matrix element $T(\Delta\Omega)$ of such a system is

$$T(X)=c_0B(G)\frac{\sin[XK]}{K}\frac{1}{(1+[2X/\ln G]^2)}\times\left\{\frac{\cos[X]}{\sin[X]}+\frac{2X}{\ln G}\frac{G+1}{G-1}\right\},\tag{21}$$

$$X=\frac{\Delta\Omega Z_a d}{2L}=\frac{\Delta\Omega d}{4K},\quad B(G)=\frac{G-1}{G\ln G}.$$

Here gain $G=\exp(2\gamma Z_a)$ (γ is a fiber loss). The matrix element has some particular limits. First, if $d=0$ (uniform dispersion along the system) we reproduce the result of Mollenauer *et al.* [17]: $T(\Delta\Omega)=(G-1)/(G\ln G)$ and because T is a constant, path-averaged model is just the integrable NLS equation. Second limit is the so-called "lossless" model [14] ($\gamma=0$). In this case $T(\Delta\Omega)=\sin(\Delta\Omega d/4)/(\Delta\Omega d/4)$. For large K the matrix element is

$$T(\Delta\Omega)=c_0B(G)\frac{\sin(\Delta\Omega d/4)}{\Delta\Omega d/4}.\tag{22}$$

Note that such matrix element presents the matrix element of the lossless model multiplied by the factor $B(G)$, where $B(G)=1$ as G goes to 1 for the real lossless model.

For a numeric computation we choose the variation of the dispersion $d=2$. Figure 1 shows the power of the true DM solitons obtained as a solution of Eq. (5) for the matrix element (21). A dashed line corresponds to the DM soliton for $K=1$ (two amplifiers for the dispersion period) and a solid line corresponds to the DM soliton for $K=40$ (80 amplifiers for the dispersion period). Dependence of the energy of DM soliton (for two different values of the averaged dispersion) on the parameter K is presented in Fig. 2. It is seen that DM soliton energy is saturated with the growth of K.

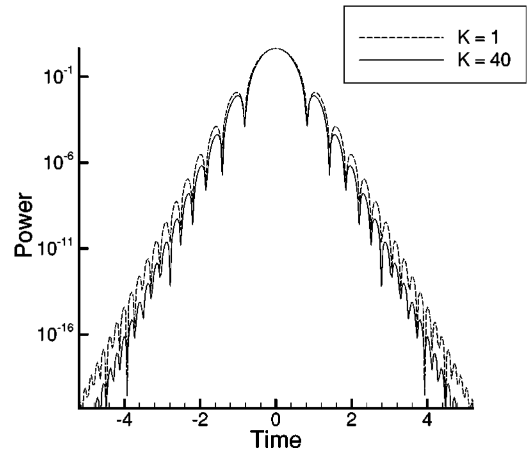

FIG. 1. A dashed line corresponds to the DM soliton for $K=1$ (two amplifiers for the dispersion period) and a solid line corresponds to the DM soliton for $K=40$ (80 amplifiers for the dispersion period) with $\langle d\rangle=0.01$.

B. Short-scale management

Next we consider the so-called short-scale dispersion management with $L\ll Z_a$ [15,16]. We choose the amplifier distance Z_a, a two-step dispersion map with dispersion compensation period $L=Z_a/J$ (km). Dispersion is $d(z)=d+\langle d\rangle$ if $k/J<z<(k+a)/J$ and $d(z)=da/(a-1)+\langle d\rangle$ if $(k+a)/J<z<(k+1)/J$, here $k=0,1,2,\ldots,J-1$ and the parameter $a\in(0,1)$ describes a position of the step. The mean-free function R defined above can be found as $R(z)=d[z-k/J-a/(2J)]$ if $k/J<z<(k+a)/J$ and $R(z)=da/(a-1)[z-k/J-(a+1)/(2J)]$ if $(k+a)/J<z<(k+1)/J$. The matrix element $T_{\omega123}$ has a self-similar structure,

$$T_{\omega123}=c_0B(G)F(a,Z,Y),\tag{23}$$

$$F(a,Z,Y)=\left[1+\frac{iY}{Z-iY}\left(1-\frac{Z}{(e^Z-1)}\frac{e^{(1-a)Z+iaY}-1}{(1-a)Z+iaY}\right)\right]\times e^{-iaY/2}.\tag{24}$$

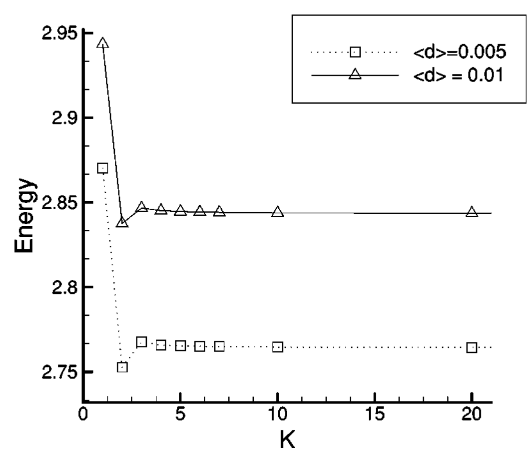

FIG. 2. Energy of the DM soliton vs parameter K for $\langle d\rangle=0.005$ (solid line) and for $\langle d\rangle=0.01$ (dotted line).

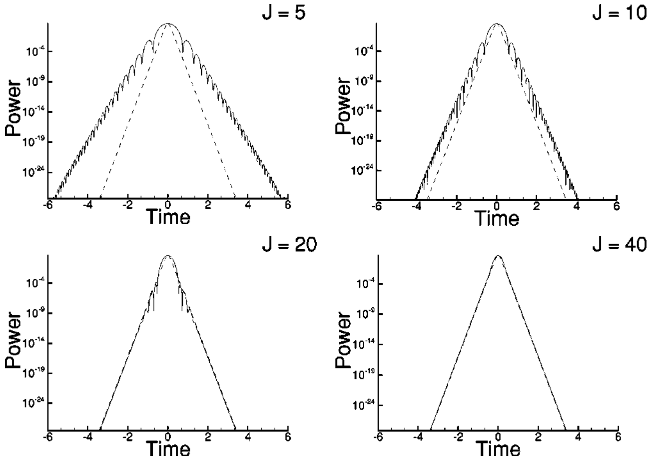

FIG. 3. The power of the DM solitons (solid line) and the fundamental soliton with the same amplitude (dashed line) for different J.

Here an amplitude B is a function of $G=\exp(2\gamma Z_a)$ only and is independent on J. A shape $F(a,Z,Y)$ is a function of the parameter a and specific combinations of $Z=\ln G/J$ and $Y=d\Delta\Omega/J$.

In the limit $d=0$ we obviously recover results of the traditional path-averaged (guiding-center) soliton theory [17–19]. One can see that with increase of J (for the fixed other parameters) the path-averaged model (2) governing DM soliton propagation converges to the integrable NLS equation with $T(0)=c_0B(G)$. It is obvious that in the limit of a very weak loss (small γ) we again obtain for T the lossless model approximation: $T_{\omega 123}=c_0\sin(aY)/(aY)$. However, increase of J (decrease of L) under the fixed characteristic bandwidth of the signal makes insignificant oscillatory structure of the kernel [20]. This means that if $T(\Delta\Omega)$ is practically concentrated in some region, then for large J the corresponding region in $\Delta\Omega$ will be larger than for small J. For the pulses with the same spectral width this will mean that T is much flatter for large J and, as a matter of fact, for large J (small L) function T can be well approximated by a value $T(0)$. As a result, NLSE model works rather well in this limit and solution (of the path-averaged model) should be close to cosh-like soliton of the NLSE. Note that although it is known that for the lossless model in the so-called weak map ($s<1$) limit [1,14,22,23] the DM soliton shape is close to cosh, this is not so obvious for system with loss and different periods of amplification and dispersion variations. This also means that all the control techniques developed for the improvement of the traditional soliton transmission can be directly used in these systems.

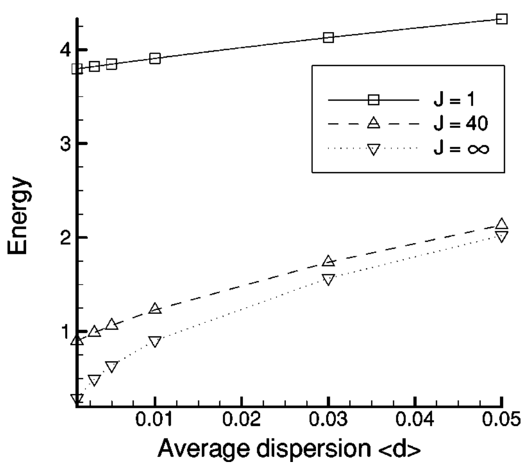

FIG. 4. The dependence of the DM soliton energy on the average dispersion for different J: $J=1$ (solid line), $J=40$ (dashed line), and for the case J tends to infinity (dotted line).

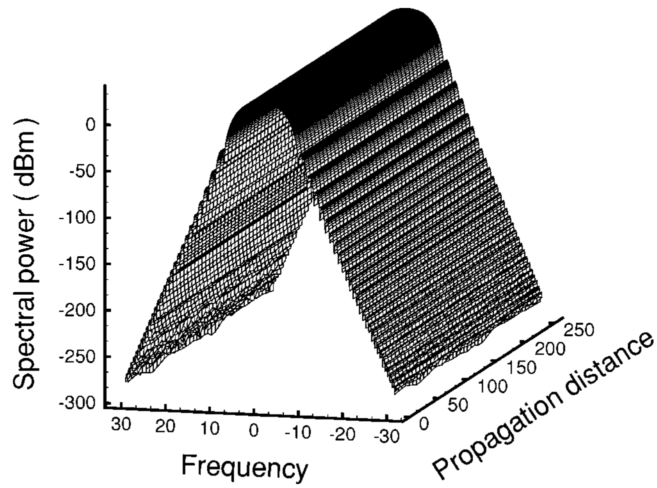

FIG. 5. The dynamics of the computed stationary solution during 40 amplifier periods with short-scale dispersion management for $J=5$.

For a numerical computation we choose the following parameters: $a=0.5$, $d=0.5$, and $\langle d\rangle=0.01$.

The power of the DM solitons (solid line) and the fundamental soliton with the same amplitude (dashed line) are plotted in Fig. 3 for different J. It can be seen that for large J the form of the DM solitons is very close to the form of the fundamental soliton. This result is an agreement with the theory presented in [5,24].

Figure 4 shows how energy of the DM soliton depends on the average dispersion for different values of J. It is seen in agreement with lossless model that the DM soliton can have a finite energy even for zero average dispersion.

Figure 5 demonstrates the dynamics of the computed stationary solution during 40 amplifier periods with short-scale dispersion management for $J=5$. One can see that the DM soliton resolved here with huge accuracy is stable and travels along the system without any radiation.

C. Conclusions

We have presented results of theoretical and numerical study of the properties of path-averaged optical soliton in double-periodic DM systems. We propose a different numerical method to compute DM solitons in the path-averaged models.

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